NAG Toolbox for MATLAB

c06ea

1 Purpose

c06ea calculates the discrete Fourier transform of a sequence of n real data values. (No extra workspace required.)

2 Syntax

```
[x, ifail] = c06ea(x, 'n', n)
```

3 Description

Given a sequence of n real data values x_j, for $j = 0, 1, \ldots, n-1$, c06ea calculates their discrete Fourier transform defined by

$$\hat{z}_k = \frac{1}{\sqrt{n}} \sum_{j=0}^{n-1} x_j \times \exp\left(-i\frac{2\pi jk}{n}\right), \qquad k = 0, 1, \ldots, n-1.$$

(Note the scale factor of $\frac{1}{\sqrt{n}}$ in this definition.) The transformed values $\hat{z}_k$ are complex, but they form a Hermitian sequence (i.e., $\hat{z}_{n-k}$ is the complex conjugate of $\hat{z}_k$), so they are completely determined by n real numbers (see also the C06 Chapter Introduction).

To compute the inverse discrete Fourier transform defined by

$$\hat{w}_k = \frac{1}{\sqrt{n}} \sum_{j=0}^{n-1} x_j \times \exp\left(+i\frac{2\pi jk}{n}\right),$$

this function should be followed by a call of c06gb to form the complex conjugates of the $\hat{z}_k$.

c06ea uses the fast Fourier transform (FFT) algorithm (see Brigham 1974). There are some restrictions on the value of n (see Section 5).

4 References

Brigham E O 1974 *The Fast Fourier Transform* Prentice–Hall

5 Parameters

5.1 Compulsory Input Parameters

1: **x(n) – double array**

$\mathbf{x}(j+1)$ must contain x_j, for $j = 0, 1, \ldots, n-1$.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The dimension of the array **x**.

n, the number of data values. The largest prime factor of **n** must not exceed 19, and the total number of prime factors of **n**, counting repetitions, must not exceed 20.

Constraint: $\mathbf{n} > 1$.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **x**(**n**) – **double array**

The discrete Fourier transform stored in Hermitian form. If the components of the transform $\hat{z}_k$ are written as $a_k + ib_k$, and if **x** is declared with bounds $(0 : \mathbf{n} - 1)$ in the (sub)program from which c06ea is called, then for $0 \leq k \leq n/2$, a_k is contained in $\mathbf{x}(k)$, and for $1 \leq k \leq (n-1)/2$, b_k is contained in $\mathbf{x}(n-k)$. (See also Section missing entity c06background12 in the C06 Chapter Introduction and Section 9.)

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

At least one of the prime factors of **n** is greater than 19.

ifail = 2

n has more than 20 prime factors.

ifail = 3

On entry, $\mathbf{n} \leq 1$.

ifail = 4

An unexpected error has occurred in an internal call. Check all (sub)program calls and array dimensions. Seek expert help.

7 Accuracy

Some indication of accuracy can be obtained by performing a subsequent inverse transform and comparing the results with the original sequence (in exact arithmetic they would be identical).

8 Further Comments

The time taken is approximately proportional to $n \times \log n$, but also depends on the factorization of n. c06ea is faster if the only prime factors of n are 2, 3 or 5; and fastest of all if n is a power of 2.

On the other hand, c06ea is particularly slow if n has several unpaired prime factors, i.e., if the 'square-free' part of n has several factors. For such values of n, c06fa (which requires an additional n elements of workspace) is considerably faster.

9 Example

```
x = [0.34907;
     0.5489000000000001;
     0.74776;
     0.94459;
     1.1385;
```

```
     1.3285;
     1.5137];
[xOut, ifail] = c06ea(x)
```

```
xOut =
    2.4836
   -0.2660
   -0.2577
   -0.2564
    0.0581
    0.2030
    0.5309
ifail =
           0
```